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Optimal design of acoustical sandwich panels with genetic algorithm

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ABSTRACT

An optimization study was performed to design a sandwich panel with a balance of acoustical and mechanical properties at minimal weight. An acoustical model based on higher order sandwich beam theory was used with mechanical analysis of the maximum deflection at the center of the sandwich panel under a concentrated force. By constraining the acoustical and mechanical behavior of the sandwich panel, the area mass density of the sandwich panel was minimized using a genetic algorithm. The sandwich panels were constructed from eight face-sheet materials and sixteen core materials, and the thicknesses of the face-sheets and the core were varied. The resulting design is a light-weight, mechanically efficient sound insulator with strength and stiffness comparable to sandwich structures commonly used in structural applications.

1 INTRODUCTION

Composite sandwich panels are widely used in weight-critical industrial applications because of the extremely high stiffness-to-mass ratio. The same high stiffness-to-mass ratio that imparts mechanical efficiency also imparts efficient transmission and radiation of acoustic noise, posing a serious problem for vibro-acoustical applications. To address this problem, engineers have attempted to identify optimal designs for sandwich panels that balance mechanical and acoustical properties. For example, Lang and Dym [1] studied the optimal acoustic design of sandwich structures by applying their previous theory of sound transmission through sandwich panels [2]. Using the pattern search method [3], various combinations of design variables were evaluated to improve the sound insulation capacity of sandwich panels. In related work, Makris et al. [4] reviewed the single-objective optimization study of acoustic sandwich panels [1], and proposed a more sophisticated optimization technique. Three optimization techniques were evaluated, including the Hooke-Jeeves pattern search method [3], the complex method of Box [5], and the flexible tolerance method [6]. Of these three techniques, the flexible tolerance method was chosen as the most suitable optimization technique. Closed-form expressions between TL and the core optimization design variables were found for a set of different skins by curve fitting. More recently, Wennhage [7] studied the weight optimization of sandwich panels, taking into account both structural and acoustical requirements. An optimization package based on the method of Moving Asymptotes [8] was used to find the set of design variables that minimize the weight of the sandwich panel under the constraints.

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The references cited above illustrate that several techniques have been proposed for this optimization problem. The pattern search method utilized in Ref. [1] did not have high directional flexibility, and convergence to a global optimum was not guaranteed. The complex method of Box [5] is a sequential search technique which has proven effective in solving problems with nonlinear objective functions subject to inequality constraints. However, when linear constraints are present, or when equality constraints are involved, the complex method of Box is not very effective. The flexible tolerance method (FTM) [6] is a constrained random search technique. In FTM, the set of violated equalities and inequalities is combined in one inequality constraint. As the search process continues, the near-feasibility limits are gradually made more restrictive, until in the limit, only feasible vectors are accepted. In the method of moving asymptotes [8], the gradients of the objective function and constraints with respect to the design variable changes must be computed. This method also has a major disadvantage: convergence cannot be guaranteed and in practical use this fact sometimes leads to unsatisfactory results.

In this study, the genetic algorithm (GA) [9-10] is utilized as the optimization technique. A fundamental difference between GAs and the random search methods is that GAs work with a population of candidate solutions to the problem, while previous algorithms work with one solution and move toward the optimum by updating this one estimate. A GA simultaneously considers multiple candidate solutions to the problem of minimizing the objective function and iterates by moving this population of solutions toward a global optimum. Research has demonstrated that GAs can be robust in identifying global solutions in multimodal design spaces, even when numerous design variables and constraints are employed. Therefore, GAs are particularly well-suited to the present optimization study.

A theory of sound transmission through sandwich panels developed previously [11] will be used in the present paper to determine what combinations of design variables yield a sandwich panel that satisfies both the sound insulation and mechanical criteria at minimal weight. The total weight per unit of area is chosen as the objective function, with the weight-averaged sound transmission loss as the acoustical constraint and the maximal deflection at the panel center as the mechanical constraint. The design variables consist of the elastic moduli, the densities and the thicknesses of the face sheets and the core materials. A numerical example is utilized to illustrate the design process. The present approach can serve as a tool to design weight-optimized sandwich panel with maximized acoustic and mechanical properties.

2 OPTIMIZATION STRATEGY

Genetic algorithms (GAs) represent a popular approach to stochastic optimization, especially when addressing the global optimization problem of finding the best solution among multiple local minima. An optimization study involves the selection of objective function, design variable and constraints, as described below.

2.1 Objective function/Fitness function

Mass is a critical property for design of aerospace structures. It affects almost every design aspect, such as required engine thrust, wing design, and the cost of the aircraft. Therefore, for this study, the total mass per unit of area of the sandwich panel is chosen as the objective function. The quantity can be calculated from the mass densities and the thicknesses of the face sheets and the core materials,

$$W = 2\rho_{skin} t_{skin} + \rho_{core} t_{core} \tag{1}$$

2.2 Design variables

Seven design variables are used in the optimization study, including the Young's modulus, the density, the thickness of the face sheets, and the Young's modulus, the shear modulus, the density and the thickness of the core material.

2.3 Constraints

Research has revealed that designs that favor the mechanical performance of the sandwich panel may simultaneously increase both noise transmission and radiation from the panel. To design a sandwich panel that is suited for load-bearing and noise minimization, both the acoustical the mechanical properties should be constrained.

2.3.1 Acoustical constraint

Makris et. al. [4] utilized an A-weighted sound transmission loss between 1000 Hz and 4000 Hz as the objective function in the optimization study. The same function will be employed here to constrain the acoustical behavior of the weight-optimized sandwich panel. Greater values of sound transmission loss indicate improved sound insulation. Therefore, the acoustical constraint is defined as

$$TLA \geq TLA_0 \tag{2}$$

where TLA is the A-weighted sound transmission loss of the sandwich panel, and TLA_0 is the prescribed acoustical constraint. The A-weighted sound transmission loss of sandwich panels will be calculated using the consistent higher-order approach developed in [11].

2.3.2 Mechanical constraint

A simple mechanical constraint was employed by considering the maximal deflection of a simply-supported 1m×m square sandwich plate under 1N force at the center of the plate. The formula for the deflection at the center of the sandwich plate is [12]

$$d = \frac{PL^3}{48B_s}\left(1 + 4k\right) \tag{3}$$

where B_s is the bending stiffness of the sandwich panel, $k = 3B_s \big/ A_c G_{eff} L^2$, and G_{eff} is the effective shear modulus of the core. Lower deflection values result from sandwich panels with greater mechanical stiffness. Therefore, the maximal deflection is constrained as

$$d \le d_0 \tag{4}$$

3 GENETIC ALGORITHM

GAs begin with a set of solutions (represented by chromosomes) called a population. In their basic form, each number of the population is represented by a binary string that encodes the variables characterizing the design. Solutions from one population are selected and used to form a new population according to their fitness - the more suitable they are, the greater chance they have to reproduce. This is repeated until a certain condition is met. The block diagram of GA optimization is illustrated in Fig. 1.

3.1 Encoding of a Chromosome

Each chromosome should in some way contain information about the solution that it represents. The most common way of encoding is a binary string. Each chromosome is represented by a binary string. Each bit in the string can represent some characteristics of the solution. Another possibility is that the whole string can represent a number. An example will be given in the following numerical study for the binary encoding of chromosomes.

3.2 Genetic operations

The most commonly used operations in GAs are currently: (1) selection according to fitness, i.e., the most promising designs are given a greater share of the next generation; (2) crossover, where portions of two good designs, chosen at random, are used to form a new design, i.e., two parents "breed" an "offspring"; (3) mutation, where small but random changes are arbitrarily introduced into a design. In addition, the number of generations and their size must be chosen, as well as the method for dealing with constraints.

3.2.1 Selection

Chromosomes are selected from the parent population for crossover. Parents are selected according to their fitness. The better the chromosomes are, the more chances they have to be selected. Imagine a roulette wheel where all the chromosomes in the population are placed. The size of the section in the roulette wheel is proportional to the value of the fitness function of every chromosome - the greater the value, the larger the section.

3.2.2 Crossover

The crossover operator allows the reproduction of new strings through the combination of pairs of strings. A simple crossover operation is performed by selecting one crossover point, and a binary string from the beginning of the chromosome to the crossover point is copied from the first parent, while the remainder is copied from the other parent.

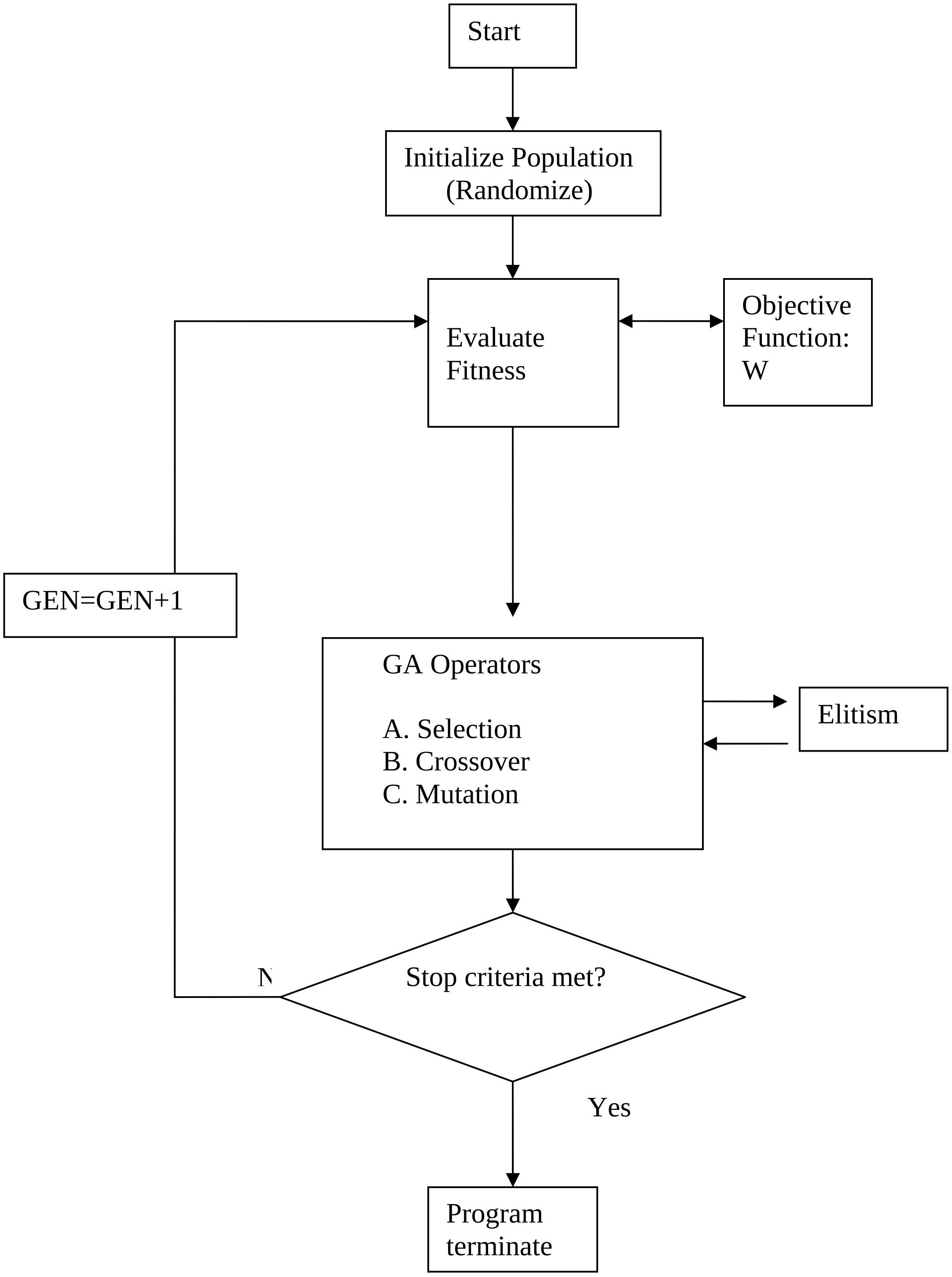

Figure 1. Block diagram of GA optimization.

$$\mathbf{1100}1011+1101\mathbf{1111} = \mathbf{11001111} \tag{5}$$

For example, in Eq. (5), the crossover point is selected between the forth and the fifth digits of the chromosome. The offspring is reproduced by copying the first four digits of the first parent and the last four digits of the second parent.

3.2.3 Mutation

Mutation is the sporadic alteration of chromosomes. Mutation is performed by inverting a bit in the binary code. The position at which the bit is inverted is randomly selected with a small probability.

$$\mathbf{11001001 \; => \; 10001001} \tag{6}$$

In Eq. (6), mutation is performed by inverting the second digit of the chromosome.

3.2.4 Elitism

When creating a new population by crossover and mutation, there is a significant chance that the “best” chromosome will be lost. Elitism is the name of the method that first copies the best chromosome (or few best chromosomes) to the new population. The rest of the population is constructed in ways described above. Elitism can rapidly increase the performance of GAs, because it prevents a loss of the best found solution.

4 NUMERICAL EXAMPLE

A numerical example is provided here to demonstrate the methodology described above. Simply-supported 1m×1m sandwich plateswere constructed from eight face sheet materials and sixteen core materials. The damping of the materials was assumed to be independent of frequency, with a fixed loss factor value of 3%. The face sheets were made from four alloys and four fiber-reinforced composites. The core was chosen from sixteen materials, including five foams, five honeycombs, a mineral wool, a plywood, a cork, a HDPE and a rubber. The properties of these materials are listed in Table 1 and 2. The thickness of the face sheets varies from 0.001m to 0.01m, and the thickness of the core lies between 0.01m to 0.1m.

Table 1. Material properties of the face sheet materials.

Materials	Density (kg/m^3)	Young's Modulus (GPa)	Shear Modulus (GPa)
Steel	7800	210	80
Aluminum	2700	71	26.7
Cast Iron	7200	130	52
Titanium	4500	110	41

Graphite/Epoxy	1600	125	3.0
Fiberglass/Epoxy	1900	56	4.2
Carbon	1600	324	1.1
Aramid/Epoxy	1500	76	2.3

The A-weighted sound transmission in the frequency range from 1000 to 4000 Hz was constrained to be greater than 45 dB. Mechanically, the maximal deflection of the sandwich panel under 1N force at the middle of the plate should be less than 0.01 mm.

In the preset optimization, there were $8(=2^3)$ face sheet materials, $16(=2^4)$ core materials, and the thicknesses of the face sheet and the core are evenly divided into $512(=2^9)$ shares. Therefore, a chromosome could look like this:

Chromosome 1	1101100100110110000000111
Chromosome 2	1101111000011110010001001

The first three digits represent the number of the material in the face-sheet database, i.e. 000 denotes steel. Similarly, the next four digits represent the core material, (e.g., 1111 represents polyethylene). The following 9 digits represent the thickness of the face sheet, with 000000000 denoting 1mm and 111111111 indicating 1cm. Similarly, the last nine digits specify the thickness of the core.

Five populations were used for each generation. Starting from these chromosomes, the genetic optimization was conducted. The fittest objective function value of each generation was plotted as a function of the generation, as shown in Fig. 2. The plot shows that the values of the objective function for the best individuals steadily decreased with generations, because elitism was employed.

Table 2. Material properties of the core materials.

Materials	Density (kg/m^3)	Young's Modulus (MPa)	Shear Modulus (MPa)
Polyurethane foam	22	0.0465	0.0166
Melamine foam	8.8	0.08	0.0286
Polyester foam	30	0.54	0.2
Cast foam	22	0.065	0.023
Plastic foam	31	0.143	0.055
Mineral wool	60	6.0	2.0
Hard Rubber	1100	2300	772
Honeycomb (1)	48	30	12.5
Honeycomb (2)	24	12	5.0
Balsa wood	144	3500	150
Plywood	700	6.0	2.4
Tricel Honey (1)	25.76	9.8	27.5 (L) 11.5 (T)
Tricel Honey (2)	38.72	14.6	47.9 (L)

			13.0 (T)
Tricel Honey (3)	47.2	19.4	63.8 (L) 14.4 (T)
Cork	180	32	5
Polyethylene (HD)	950	1000	0.31e9

After 80 generations, the results became stable and additional generation did not provide more optimal solutions. Finally, we obtained a sandwich panel, constructed from 1.3-mm-thick titanium face sheets and 72-mm-thick honeycomb core with a density of 24kg/m^2. The results are listed in Table 3.

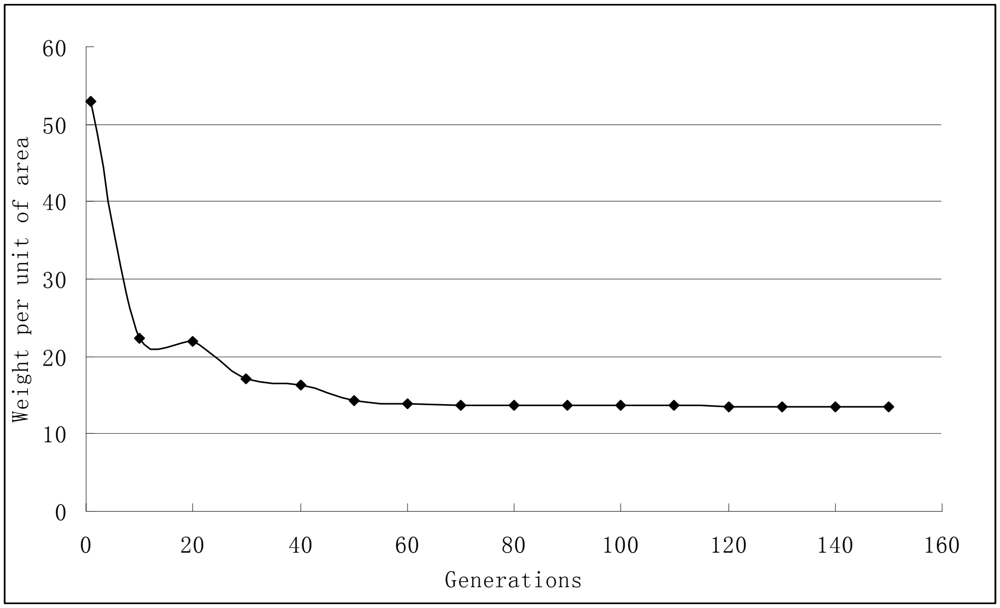

Figure 2. Variation of the fitness functions with different number of generations.

The total mass per unit of area of the sandwich panel is 13.58 kg/m^2, and the weight sound transmission loss was 45.05 dBA. Under 1N force at the middle of the panel, the maximum deflections was 0.007mm. Other designs of the sandwich panel can be obtained by changing or adding more constraint values, such as the cost of the sandwich panel.

Table 3. Properties of the optimized sandwich panel.

	Mass per unit of area (kg/m^2)	STLA (dBA)	Maximal deflection (mm/N)
Optimized Sandwich panel	13.58	45.05	0.007

5 CONCLUSIONS

A weight-optimized sandwich panel was designed using a genetic algorithm, taking into consideration both the acoustical and mechanical properties of the panel. Eight conventional face sheet materials and sixteen core materials were chosen as base materials to design the sandwich panel. The consistent-higher order prediction was utilized to constrain the sound insulation of the sandwich panel, while the maximal deflection of the sandwich panel was selected as the constraint for the mechanical stiffness.

The analysis demonstrated that the GA approach could be used to minimize the mass per unit of area of the sandwich panel by varying the material properties and thicknesses of the face sheets and core materials, while simultaneously fulfilling acoustical and mechanical constraints for the sandwich panel. The consistent higher order formulation yielded acoustic predictions and was compatible with the genetic formulations. The present approach can be used as a practical design tool to produce practical sandwich panels with superior combinations of both sound insulation and mechanical properties.

6 ACKNOWLEDGEMENTS

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